PHYS 301
Electricity and Magnetism

Dr. Gregory W. Clark

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Today!

- Electric fields
  - ➢Gauss' law
  - ➢Conductors
  - ➢Electric potential

Electrostatics
[Source charges at rest]

- **Basic problem**:
  Find forces on test charge due to source charges
- Superposition Principle holds for forces and vector fields

The Electric Field:

For a single point charge: $\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{\mathscr{r}^2}\hat{\mathscr{r}}$

For a differentially small point charge: $d\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}}$

For continuous charge distribution: $\vec{E} = \int d\vec{E} = \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}}$

where $\varepsilon_o = 8.85\times10^{-12} C^2/Nm^2$

Gauss' Law:

Integral Form:

$$\oint_{surf} \vec{E}\cdot d\vec{a} = \frac{q_{encl}}{\varepsilon_o}$$

Differential Form:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_o}$$

Note:

$$q = \int_{vol} \rho(\vec{r}\,')d\tau$$

Electric Potential

ELECTROSTATICS

- For any *static* charge distribution,

$$\vec{\nabla} \times \vec{E} = 0$$

$\therefore$ we can express $\vec{E}$ as the gradient of some scalar function:

$$\vec{E} = -\vec{\nabla} V$$

tradition!

Direction of field is in the direction of ***decreasing*** *potential*

Electric Potential

ELECTROSTATICS

- Units: $\vec{E} = \vec{F} / q \quad [N / C]$

$$\partial V = \partial r \left| \vec{E} \right| \quad [Nm / C \equiv V]$$

Volt!

- Obeys principle of superposition - just like $\vec{E}$

- Potential ≠ Potential energy (but is related)

- Potential differences are physical;
  not absolute potentials

$$V(P) = -\int_{\vartheta}^{P} \vec{E} \cdot d\vec{l}, \quad \text{where } \vartheta = \text{ref. point}$$

Fundamental Equations of Electrostatics

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_o \qquad \vec{\nabla}\times\vec{E} = 0$$

permits $\vec{E} = -\vec{\nabla}V$

- In terms of potential:

$$\Rightarrow \quad \vec{\nabla}\cdot(-\vec{\nabla}V) = -\nabla^2 V = \rho/\varepsilon_o$$

$$\nabla^2 V = -\rho/\varepsilon_o \quad \xrightarrow{\text{if } \rho = 0} \quad \nabla^2 V = 0$$

Poisson's equation — *LaPlace's equation*

Electric Potential

ELECTROSTATICS

- The workhorse of electric potential looks a lot like its electric field counter part:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\frac{q}{\mathscr{r}}$$

point charge

continuous charge distributions

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\rho(\vec{r}\,')}{|\vec{\mathscr{r}}|}d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\sigma(\vec{r}\,')}{|\vec{\mathscr{r}}|}dA'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\lambda(\vec{r}\,')}{|\vec{\mathscr{r}}|}dl'$$

relative to what?

infinity!